Changing perspectives on inclusive mathematics education: Relationships between research and teacher education

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This paper focusses on the challenges associated with teaching in inclusive mathematics classrooms and the kinds of experiences that may contribute to the development of knowledge about teaching mathematics in ways that respect student diversity. More particularly, we examine issues related to preparing teachers to include learners with disabilities within mainstream mathematics classes. Recognising the need to adopt a view of difference as difference not deficit, we present an approach which involves building collaborative research partnerships in which teachers and researchers design and analyse inclusive scenarios for mathematics learning. Using an example scenario in which a group of deaf and hearing students explored the mathematical concepts of symmetry and reflection, we point to how this process involved us in re-signifying our views of mathematics and its teaching, in order that we might "hear" and recognise mathematics even when it is expressed in forms that differ from the conventional.

Key words: Inclusive mathematics education, collaborative research, students with disabilities, deaf students, professional development, teacher education.

In recent years, the educational paradigm in Brazil has experienced large changes. One of these relates to the growing influence of political and social movements that defend inclusive education and schools that contemplate the diverse needs of all students, without any kind of segregation. According to UNESCO (2009), inclusive education is 'an ongoing process aimed at offering quality education for all while respecting diversity and the different needs and abilities, characteristics and learning expectations of the students and communities, eliminating all forms of discrimination' (p. 18). Inclusive schools are those which see difference as a factor which enriches the educational process rather than as a deficiency that impedes learning. They aim to involve all learners in quality learning experiences and to empower them to become active participants in a more equitable system.

Inclusion brings questions of social justice and equity into play, signaling the need for those entering and working in the teaching profession today to 'understand the historical, socio-cultural and ideological contexts that create discriminatory and oppressive practices in education' (Ballard, 2003: 59). Ballard points to the isolation and rejection of disabled students as one area of injustice and there can be little doubt, at least in the Brazilian context, that in the policies associated with the move towards inclusive schools, it is learners with disabilities and those previously catered for within the separate system of special education whose education is undergoing a series of particularly substantial changes.

In our country, the political policies related to the process of including students with disabilities have resulted in a significant increase in their presence within mainstream schools, with statistical data from the most recently available school census showing an increase of 234% between 2003 and 2010. At the same time, these policies of inclusion have been associated with taking the educational community out of the 'comfort' zone and, amongst the many uncertainties, insecurities and conflicts that the actors in these communities are facing, questions related to curriculum demands and pedagogical actions have a central role as too does the fundamental issue of how our (different) experiences of the world mediate the ways we learn. In the changing context of the Brazilian school system, how to ensure learners have access to an education which respects the particular ways in which they experience and participate in the world has become a present and pressing issue.

It is within this context that our research programme aiming to investigate and enable the mathematical practices of learners with disabilities was created in 2002. In this paper, we intend to address the question of preparing teachers to work in inclusive mathematics classrooms. We begin by considering the challenges associated with a teacher education that emphasises social justice and equity, focusing on the problems that need to be addressed and the models that might be employed. We then concentrate on the particular model that we adopt, a model based on collaborative research, in which we combine our expertise as teachers and researchers to reflect upon how we might create a more inclusive school mathematics while simultaneously learning about how different students appropriate mathematical knowledge.

Preparing mathematics teachers to work in inclusive classes

Our research was motivated in the first instance by a grass-roots reaction of mathematics teachers concerned about how to include groups of learners in their classrooms who had previously been educated separately. As they sought for guidance from the research community, these teachers expressed many concerns. Working with students with disabilities was not something that had been addressed in either initial or in-service education courses, leaving them feeling ill-prepared and uninformed about working with students who do not see with their eyes, who talk with their hands, who have particular ways of experiencing cognitive processes such as memory or attention, or, to paraphrase Smith *et al.* (2009), who have other 'interesting ways to be alive' (253). This lack of preparation left them with deep insecurities about what inclusion actually means in the context of mathematics education.

Such insecurities are understandable. If we are serious as a society in general, and as a community of mathematics educators in particular, to ensure that we offer quality learning experiences to all our students, we need to engage in a process of re-signifying teaching, learning, assessment, the subject matter we teach, and the structures and functions of the systems we teach in. This makes teacher education a critical issue. Indeed, Capellini and Rodrigues (2009), in their study of the challenges associated with creating inclusive schools identified by 423 Brazilian teachers, concluded that for inclusion to become more than ideology, investment in initial and in-service teacher education is required, but that '[U]nfortunaely, there are still no teacher education programmes for teachers that focus on knowledge of the disabilities that affect various children' (359).

This finding is not unique; it echoes the results of similar investigations over the last twenty years (Goffredo, 1992; Manzini, 1999; Sant'Ana, 2005). Nor is it specific to Brazil: in a recent survey into teaching and learning undertaken by the Organization of Economic Co-operation and Development, the teaching of students with “special learning needs” was the aspect most frequently cited by teachers in the 23 participating countries as an area in which they desired professional development opportunities (OECD, 2009).

Returning to the local context of our own work, it would seem that the Brazilian mathematics teachers who were seeking guidance about creating classroom cultures which respect the diversity of the students within them are not alone; they are part of a global trend. While professional development programmes which specifically address students with disabilities might be rare, as Healy and Powell (2013) indicate, the issue of preparing teachers for equity is not in itself a new one. Its importance in pre-service courses has long been stressed, especially as prospective teachers tend to have limited experiences of interacting with cultures outside of their own (Grant & Secada, 1990).

As far as the preparation of mathematics teachers is concerned, many countries have seen a shift from models based on training to those involving more practice-based professional development over the last 25 years. Matos, Powell and Stzajn (2009) associate this shift with a move from seeing learning as a process of individual acquisition of knowledge to understanding it as the appropriation of forms of participation in social practices. This shift is more recent in Brazil, with Ferreira (2006) describing how the time devoted to theoretical considerations of mathematical content in pre-service mathematics education courses continues to far outweigh the time dedicated to issues of practice, and how in-service courses for practicing teachers tend to be directed at overcoming previously determined “deficiencies” amongst the targeted participants (Ferreira, 2006: 149). Freitas and Fiorentini (2011), too, observe how, in Brazil it is common for mathematics teacher education courses to ‘still maintain a teaching practice that values orality, explanation, repetition of long lists of exercises, and distribution of a systematic, formalized, and already built knowledge’ (195). Nonetheless, there is evidence, at least in some parts of Brazil, of the emergence of approaches to teacher education that are

> based on reflection, collaboration, and inquiry and focus on practices of teaching and learning mathematics in different contexts. They are founded on the assumption that teachers can construct knowledge in and from practice. Their knowledge and their socio-cultural practices are the starting and arriving points of continuous professional development (Bednarz, Fiorentini and Huang, 2011: 234).

The examples of Brazilian mathematics teacher education approaches explored by Bednarz, Fiorentini and Huang are not specifically aimed at addressing issues of social justice and equity, nor do they focus on the challenges associated with the inclusion of disabled learners, but they do contemplate some elements that have been identified as essential for the promotion and implementation of equity principles in the classroom. For example, Battey, Kafal, Nixon and Kao (2007) list as essential inquiry, collaboration, a focus on classroom practice, and consideration of the larger social and political context. They suggest that to achieve more equitable mathematics classrooms, teachers need to become active participants in researching and interpreting their students' learning, and should engage in the processes of reflecting on their beliefs about the mathematics that different students do and how they do it.

Healy and Powell (2013) suggest an important step in involving teachers in such activities is to move away from views of difference as deficit. This position was also emphasised in a recent review of European research into teacher education and inclusion, which concluded that any teaching is likely to be ineffective where the dominant belief system is one that 'regards some students as being 'in need of fixing' or worse, as 'deficient and therefore beyond fixing'' (European Agency for Development in Special Needs Education, 2010: 30). For Healy and Powell, change in teachers' perceptions of the mathematical agency of students from marginalized groups requires a focus on what these students can do, rather than lamentations of what they do not. That is, they argue that, instead of attempting to determine "normal" or "ideal" achievement and positioning those who deviate from supposed norms as problematic and in need of remediation, attention should be directed to how students' mathematical ideas develop differently and the pedagogical strategies appropriate to support these developmental trajectories. Their view is that participation in research studies and in research-based teacher development programmes offers a possible means of promoting such a shift. The research programme that we describe in the next section, and in which we are both involved, is an example of such an enterprise.

Collaborative research into inclusive school mathematics scenarios

Our research programme, *Towards an Inclusive School Mathematics*, has three main aims: to investigate forms of accessing and expressing mathematics that respect the diverse needs of all our students; to contribute to the development of teaching strategies which recognise this diversity; and to explore the relationships between sensory experience and mathematical knowledge. The project involves the design and analysis of inclusive scenarios for mathematics learning, through a collaborative process resulting from a series of iterative interactions between researchers, teachers and students. The design of the learning scenarios involves the simultaneous development of the material and/or digital tools, tasks and teaching interventions, aimed at enabling multiple ways of interacting with mathematical objects and relations. Mathematical ideas are hence presented through media such as colour, sound, music, movement and texture, in order that they might appeal to different sensory canals, to the skin, to the ears and to the eyes.

In the rest of this article, we intend to exemplify our approach to illuminating and supporting the practices of mathematics students with (and without) disabilities by considering the design and investigation of a particular scenario, conceived to enable deaf and hearing students to explore the geometrical transformation reflection. We were both involved in this project. One of us (Lulu) is a member of the research community, and a lecturer on a post-graduate programme for Mathematics Education. The other (Heliel) is a practising mathematics teacher who joined the project while working in a public school in a town on the western outskirts of the metropolitan conurbation of São Paulo. Like most of the teachers involved in the project[1], prior to joining, Heliel had received no specific preparation related to inclusive education or to teaching mathematics to deaf students. We worked together on the design and analysis of the symmetry and reflection learning scenario, which served as the focus for Heliel's Masters Dissertation (Santos, 2012).

Following the completion of this dissertation, we video-taped a conversation in which we reflected together on the impact of participating in this study on our professional identities as both teachers and researchers. Below we present the opening words of this discussion:

> Heliel: *First I want to say how important this whole experience was for me. It was like a parting of the waters, a defining moment, in the question of inclusion and the question of teaching and learning mathematics. Sharing information with all*

[1] More information about the programme *Towards an Inclusive School Mathematics* and the other members of the research team is available on the site www.matematicainclusiva.net.br.

the others in the group was fundamental for me in becoming the teacher that I am today.

In order that we might identify what motivated this opening statement and explore how collaborating in the design and analysis of scenarios for inclusive mathematics education is shaping our views on inclusion and mathematics, we intend to use the symmetry and reflection learning scenario as a kind of generic example. We will outline the tasks proposed and the tools by which they were to be mediated, as well as presenting a snapshot of the students' activities as the tools and tasks were put into action. Alongside these descriptions, we will offer extracts from our conversation, privileging the teacher's voice, as we attempt to interpret how a collaborative research project can contribute to deepening our knowledge about teaching mathematics in inclusive settings. First, though, we begin by presenting the methodology used in the research programme as a whole.

Methods

As members of different communities, we bring different concerns and different priorities, to the studies we jointly construct. We believe that the fusing of research and teaching perspectives enriches the investigative process, allowing us to engage both in constructing theoretical interpretations of the processes of mathematical learning and with the practical demands of teaching school mathematics. The methodology that we use is also hybrid. It combines aspects drawn from co-generative inquiry, a kind of participatory action research, with methods associated with design experiments. To conduct co-generative inquiry, all participants work together to construct knowledge through a process of collaborative communication (Greenwood and Levin, 2000). In a co-generative study, the inquiry process is linked to action in a given context – in this case Heliel's school – and involves participants in re-signifying the practices within this context. Design experiments, on the other hand, can be described as 'iterative, situated, and theory-based attempts simultaneously to understand and improve educational processes' (diSessa and Cobb, 2004: 80). They focus on interactions which occur within conceptually rich scenarios, specifically crafted to optimise the chances that learning will not only occur, but will occur in an observable form. Designing the learning scenario is hence a complex task, requiring the identification of the particular challenges associated with learning the mathematical topic in question, in developing and testing hypotheses about how best to engender the intended learning, in fine-tuning the theoretical models behind these hypotheses so that they can be operationalised in the given context and in interpreting the kinds of activities and behaviour which indicate that learning is taking place. This complexity demands real and sustained collaboration between researchers and teachers.

In general, the design and analysis of each learning scenario passes through the same set of procedures. The first step involves an initial interview with the teacher participant(s), who are asked to describe their motivation for participating and what they hope to learn/gain. Since most of the participating teachers are also post-graduate students, it is customary for one teacher to adopt a lead role in any given scenario, with at least one member from the research community as principal partner. It is the lead teacher who normally determines the mathematical content to be explored. At the end of each study, individual interviews are also conducted and participants are asked to provide reports reflecting upon what they consider to be the most significant moments. Project meetings devoted to the design and modification of learning situations and tools are held regularly. In these meetings, existing research related to the mathematical content is discussed. Almost invariantly, this existing research brings evidence mainly of the practices of learners without disabilities. In addition to consulting research studies specifically addressing the mathematical concepts chosen for investigation, these meetings also represent an opportunity to discuss the theoretical influences that are mediating the view of mathematics learning that underpins the research programme as a whole.

Major design decisions are registered during or immediately following these meetings and versions of the developing research instruments (tools and tasks) are carefully recorded. Data is also collected by the participating teachers and researchers during the implementation of the designed learning scenarios in the partnership schools. The research sessions are video-recorded and, in some cases, research notes of other team members also present during the implementation are also available. All students' work elaborated during these implementations is collected. Hence, our data involves multiple sources and multiple observers. The analyses we undertake are also multifaceted. We attempt to interpret the embodied, semiotic and cultural processes by which students make sense of the mathematical objects they encounter and, using the same data, we also reflect upon our own interventions as teachers and upon the beliefs and attitudes that motivate them. In this way, we hope to contribute both to debates about how mathematics learning occurs and what teaching knowledge might support these learning processes.

Expressing and exploring symmetry and reflection: an example scenario.

The example scenario we present in this paper illustrates our attempts to understand the challenges associated with learning mathematics in a bilingual setting, in which teaching is conducted in both Portuguese and Libras, the sign language used by the deaf community in Brazil. The eight participating students, five of whom were deaf and three hearing, were all from the same class of a mainstream school in the municipal of Barueri, São Paulo. In this school, although not all the teachers are bilingual, interpreters are present to assist in the classes of the monolingual teachers or those with only limited fluency in Libras. As highlighted above, the first steps in the process of design involve choosing the student group and the mathematical content. Our practice is to give the teacher the first say in determining these factors. As it turned out in this case, the decision to work with symmetry and reflection was in reality a joint one, influenced by both of us. Heliel was keen from the beginning to focus on an idea from geometry covered in the school curriculum for the 7th grade. Amongst the available possibilities, Lulu suggested the choice of reflection and symmetry, as she had previous experience in investigating the appropriation of this concept by blind learners (Fernandes and Healy, 2007) and by learners without disabilities (Healy, 2002).

We were unable to locate any literature specifically focussed on the learning of symmetry and reflection by deaf and hearing students working in a bilingual setting. The literature concerning mathematics learning of deaf students mostly emphasises their performance in relation to arithmetic tasks (see, for example, Bull, 2008; Kelly, Lang & Pagliaro, 2003; Nunes, 2004; Pagliaro, 2006). Nonetheless, using the findings associated with this literature as well as research into bilingual approaches to literacy learning for the deaf, we used as one of our design principles the importance of taking into account the range of sensory modalities available to the students (Mayer & Akamatsu, 2003). This was one of the reasons that we chose to structure the learning activities in our study around a digital microworld, *Transtaruga* [2] (Figure 1), which offered different modes of interacting with geometrical ideas (visual, dynamic and symbolic). Using the vision of microworlds first presented by Papert (1980), *Transtaruga* was also conceived to explore the potentiality of using digital tools to create a mathematics which has more compatibility with the human body than conventional school mathematics. *Transtaruga* is a multiple-turtle geometry microworld written in the Imagine version of the programming language Logo, which was created by adapting the *Turtle Mirrors* microworlds previously used in research with hearing students (Hoyles & Healy, 1997; Healy, 2002).

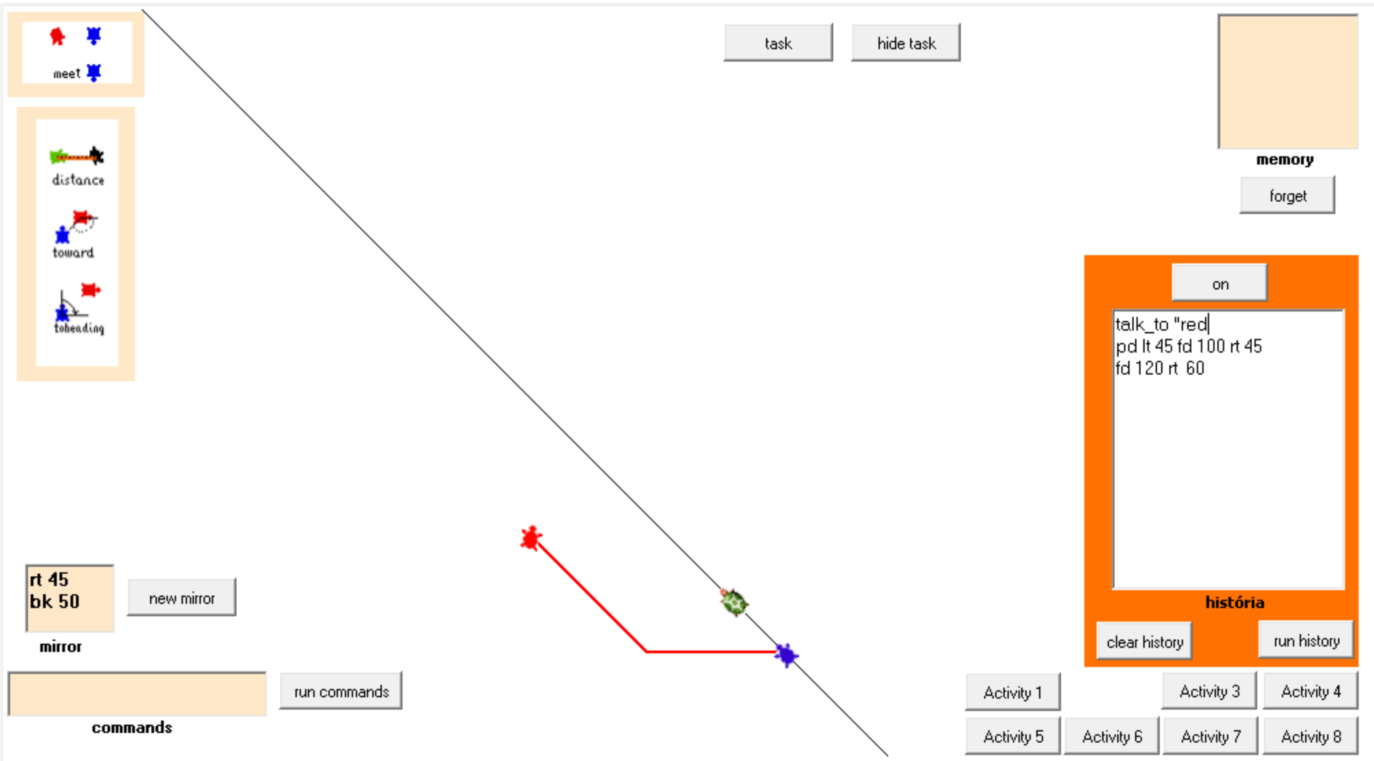

Figure 1: A page from the *Transtaruga* microworld[3]

A set of eight tasks was developed for exploration within the microworld during three research sessions, each of approximately 90 minutes. These tasks involved producing symmetrical trajectories, exploring the relationships between a turtle and its image under reflection and reconstructing missing axes of symmetry. To negotiate the task demands, the students were introduced to a limited subset of turtle

[2] Borrowing "*trans*" from the Portuguese word for transformation and "*taruga*" from tartaruga or turtle in Portuguese.
[3] We have chosen to translate the tools and commands into English to facilitate the readability of this article. In the original version, the the tool names and commands were presented in Portuguese..

geometry commands: **fd** (forward), **bk** (backward,), **rt** (right), **lt** (left). **pd** (pendown) and **pu** (penup) and shown how to activate different turtles by clicking on them. Four additional microworld tools were also presented: **meet**, which constructed a point (turtle) where two turtles intersect; **distance** which measured the distance between two turtles, displaying the value in the **memory** box; **toward**, which measured the angle one turtle should turn to point towards another, again with the result displayed in the **memory** box; and **toheading**, which measured the angle one turtle should turn so as to which measured the angle one turtle should turn so as to face the same direction as another.

We felt that the dynamic visuality of the microworld tools would be particularly important in supporting the participation of the deaf students. Indeed, how we might exploit the centrality of the visual field in the reasoning processes of deaf students has become an important focus of our attempts to build learning scenarios fine-tuned to their strengths:

> Heliel: *At the moment of thinking of any activity, we need to think about how the students are going to make sense of the mathematics. In the case of deaf students, of the senses, they have a tendency to prefer the visual, a fluency in their visual thinking. Perhaps this is natural, given that the language they use involves visual-gestural communication. Now, I think it is pertinent that we always think of the visual question with deaf students*.

The students were divided into pairs to work on the activities, with each pair positioned in front of one of four laptops, arranged in a semicircle so that, during whole group discussions, the signs of those using Libras could be easily seen by all the participants. Two pairs were comprised of only deaf students, one pair was made up of two hearing students and the fourth pair had one deaf and one hearing member. The hearing student in this pair spoke some Libras and the deaf student was partially oralised. None of the eight students had had any previous experience with the Logo programming language, nor were digital resources a feature of their regular mathematics lessons. Each laptop had a webcam and we were able to capture all on-screen activity as well as the faces of the students and the discussions between them while they worked on the tasks. In addition, three other video cameras were positioned to film all the interactions of the pairs using Libras. During the group discussions which followed the completion of each task, the student pairs were invited to connect their laptop to a data protector in order that they might share the strategies that they had used. As well as the eight students, both of us along with three other researchers and an interpreter participated in the research sessions. The researchers all adopted, at different moments, the roles of observers, technicians and teachers, although in the main we two (Lulu and Heliel) assumed the principle teaching roles. In particular, during the group discussions, Lulu acted as teacher, with Heliel and the interpreter translating her spoken interventions into Libras.

From this description, it should be clear that although we were collecting data in the school setting, it was hardly a regular classroom situation. The room was full of recording equipment, necessary to collect students' interactions when visual-gestural expressions as well as spoken ones are to be analysed. Only a sub-group of the 7th grade class of 25 students participated in the research sessions. The sessions lasted 90 minutes rather than the usual 50. The physical layout of the class had been altered and there was a much greater emphasis on student explanation than usual. These differences result from the hybrid nature of our methodology: although data is collected in the field, in the schools in which the teachers amongst us teach, the learning scenarios tend to have a structure that differs from the teaching situations which usually occur in their school contexts and are not as tightly governed by institutional and curriculum constraints. Differences are necessary as responding to our research questions requires very detailed data. One consequence, however, of this structuring of the scenario as a research setting is that we lose a certain degree of authenticity. As teachers, we know that in order to apply the lessons we might learn from these sessions in regular classroom practice, some adaptions will have to be made. Nonetheless, we think that what we are able to glean about the processes of learning more than compensates for the limitations. In particular, as we will attempt to show, the possibility of analysing student interactions in the degree of detail permitted by the way we collected the data brings valuable contributions to our developing teaching knowledge about including both deaf and hearing students in a bilingual approach to mathematics learning.

In the symmetry and reflection scenario, the first tasks given to the students involved constructing symmetrical trajectories – that is, given the set of commands used to construct a trajectory of the red turtle, the students' task was to construct a trajectory for the blue turtle which would leave a trace symmetrical to that of the red turtle in relation to the axis of symmetry. Both the red and blue turtles started from the same location. Figure 2 shows a solution to one of these tasks. The idea was that the students would notice that producing symmetrical trajectories requires using the same input to the commands which control the distance travelled by each turtle (the commands `fd` and `bk`) and swopping the direction of the turns (that is swopping `rt` for `lt` and vice-versa).

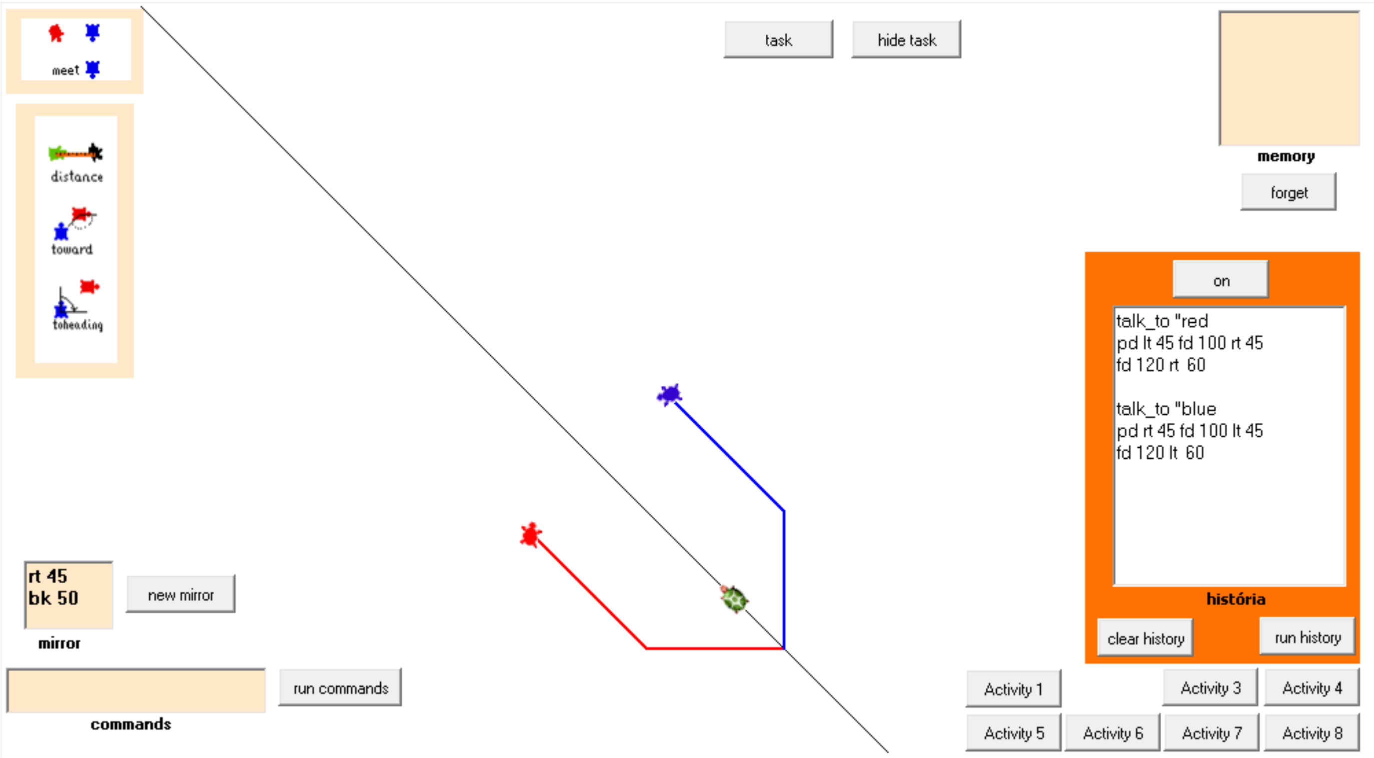

Figure 2: Constructing symmetrical paths

All eight students successfully completed the symmetrical trajectory tasks and were able to identify the relationships between the two turtle paths, although there were some differences in the strategies used by the pairs composed only of deaf students in comparison to the other two pairs. While for the hearing students the function of the Logo commands was quickly appropriated, not surprising perhaps since the Logo commands were abbreviations of familiar words, the deaf students needed to find ways to associate the particular movements of the commands with the letters of the Logo code. They did this by re-enacting the commands themselves, using, for example, their fingers to imitate the turtle movements. A second language-related difference concerned the signs used to express the mathematics content. The spoken language terms “symmetry” (or *simetria*) and “reflection” (*reflexão*) were familiar to the hearing students, but there are as yet no widely used signs in Libras for either term. Rather than being a problem, the negotiation of shared signs for symmetry and reflection became part of the solution strategy. As the deaf students became more confident in articulating the relationships between the turns made in symmetrical paths, they not only successfully produced the symmetrical trajectories, they also began to offer possible signs to represent the process of reflecting. They eventually agreed upon a sign offered by Daniel, which involved placing his hands so they were facing each other in space, and then moving both his hands together as if the space was flipped over an imagined plane (Figure 3)[4].

Figure 3: Daniel's sign for reflection

As we watched the video (many times) during the process of data analysis, we were struck by the extent to which his visual-gestural expression models the process of mapping a three-dimensional space onto itself, which mathematically characterises the reflection function. We also realised that as teachers we have tended not to 'hear' mathematics when it in expressed through body movements instead of in words or conventional symbols. We now believe that attending more closely to the mathematical expressions of those who communicate in sign languages may not only contribute to including deaf learners in mathematical practices, it may also open windows on the role of gestures and visual expressions in mathematics learning more generally. This in turn challenges us to rethink our view of mathematics:

> Heliel: *Through these experiences I became more aware that mathematics is not always the formal answer we are expecting. […] We have to pay more attention to the ways the students are expressing, to the signs they use, and that, even if these are not always conventional they can still be valid mathematically*. […] *Today, without doubt, I see these alternatives as mathematics. I think it is possible for us to teach, and to learn, mathematics through sound, through the visual question. A little while back I would have been a bit, a bit prejudiced in this respect. I don't know if I would have looked at say MusiCALcolorida*[5] *and thought that in fact I*

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[4] For more details on the embodied nature of the sense-making processes of the deaf students during this task see Healy (2012).

[5] *MusiCALcolorida* is a digital calculator which represents numbers colourfully and musically as well as numerically, more details about its use with blind students, deaf students and students without

was working with equivalent fractions, with recurring decimals. I don't think I would have had this vision.

A related point concerns the ways mathematics was expressed in the microworld – which also differed from the conventional school mathematics representations familiar to Heliel and his students. In order to solve the first set of tasks, all the students, to a greater or lesser degree, imagined at some point making themselves the movements they observed on screen. Although not originally planned, we decided to take advantage of this tendency and, as part of the whole group discussions, we developed a practice in which the students themselves acted out microworld activities, assuming the roles of the turtles. Figures 4, 5 and 6 show three moments from the beginning of the third session, during which Pedro and Daniel adopted the roles of a turtle and image, and enacted two properties of the reflection transformation: the equidistance of point and image from the axis of symmetry and the perpendicular relationship between the segment joining point and image and the axis.

Figure 4: Pedro and Daniel as turtle and image

Figure 5 Meeting on the axis after moving forward the same distance

Figure 6: Turning 90° to face along the axis

The practice of acting out the role of the turtle turned out to be an efficient way of enabling communication between those using sign and those using spoken language as well as permitting the students to experience bodily the mathematical properties

disabilities are available in Healy and Kynigos, (2009) and Fernandes *et al*. (2011). Both this application and *Transtaruga* are available for download on the project website.

associated with symmetrical figures and reflection. This practice became a regular feature of the learning scenarios – and one which contributed to our reflections about learning mathematics:

> Heliel: *Transtaruga is a microworld that I have fallen in love with, turtles have become my favourite animals and I would like them to spread throughout Barueri (laughs). I really appreciate how it enabled not only the interaction with the software, but it invites us to imitate. This really caught my attention, my imagination, this question of imitating what was occurring on screen. I perceived that when we imitated, when the students acted out the role of the turtles this has a strong contribution to their sense of the mathematical concepts that were involved in the tasks.*

As the study progressed and we became more aware of how we might exploit the practice, common in the deaf community, of negotiating shared signs in conjunction with inviting students to embody mathematical actions, the importance of encouraging all the students to express mathematics in their own terms became a more central aspect of the scenario design – and a concern in reflections about inclusive pedagogic practices:

> Heliel: *It had always been part of my practice to ask students to share their answers, but I didn´t give so much attention to getting them to explain how they arrived at these solutions, how they had elaborated them. After our work, in our data collection, we concentrated a lot of how the deaf students expressed their ideas, and now, since then, I ask them to do this much more. Especially the deaf students. I ask them to stand up and explain, next to the interpreter, so the deaf students can follow, but also the others as well*.

Valuing students' voices, regardless of whether they are expressed as spoken words or visual-gestural signs, is not only a way of supporting learners to engage with school mathematics. It is also a way of demonstrating how they are valued as members of the learning community, of publically acknowledging how different perspectives and different ways of communicating mathematically enrich school mathematics. This too we see as an essential part of teaching knowledge for inclusive education:

> Heliel: *Behind all this, I think, is a question of self-esteem. For me this was very clear in the position expressed by one of the deaf students, Elaine. Remember, after one of the first sessions. What she said really impressed on me. When she said that the deaf student also wants to learn, is also capable, also wants to*

progress. She said that, as a deaf student, she also wants to go to university. She left a strong impression on me. And when, for example, the deaf student is invited to go to the front, to explain the material, both the hearing and the deaf students, they feel it, they perceive it, that they are included, that they are part of the group and that they have as much potential as any other student.

Research-led teacher education or learning through research: A final reflection

Our aim in this paper has been to reflect upon how participating in a collaborative research project with the aims of investigating diverse forms of accessing and expressing mathematics contributes to the development of teaching knowledge appropriate for a more inclusive school mathematics. Attending to the points raised in the teacher's voice in relation to the example scenario, we have argued that the process of design, analysis and re-design impinged upon our attempts to fine-tune tasks so that students could use the forms of reasoning that make most sense to them, influenced our views of what counts as a mathematical expression, and emphasised particular strategies by which we might empower students whose voices have previously been marginalized, unheard or even ignored to share their mathematical practices. Using a lens focussed on difference rather than deficit, these reflections lead to a view of the inclusive classroom as one in which all – teachers and students – benefit.

> Heliel: *When a class has a student with some kind of disability, a deaf student, a blind student, or a student with some other disability, we might at the first moment think this will be a great burden, a cross that I am going to have to bear, because I am going to have to prepare different versions of the activities. I am going to have to stop and wait for the interpreter to translate. But looking back, you see it's actually the opposite. It's really positive when you have this kind of student, you learn a lot with this experience […]Also, I believe that for the students without disabilities, they learn a lot with the other student, they learn a lot in the class because when the teacher diversifies for the disabled students, when you explain once, then you explain again for the interpreter or you create a different approach to include a particular student, then the students without disabilities, the so-called regular students, they have a different vision about the ideas that are being explained. If they didn't understand very well, they have a new chance. So I think, in every way, everyone, everyone ends up gaining a lot with inclusion.*

As a final reflection upon the data presented in this paper, we wonder if applying the label of research-led teacher education to our particular approach is entirely adequate. Although we are members of different professional communities, the project involves us in crossing the boundaries between teaching and researching and in this sense, though we have highlighted the teacher's voice in this paper, we feel that we have both developed as teachers and we have both developed as researchers. We are a little worried about referring to the participation of teachers in collaborative research projects in terms of exercises in teacher education. There is a danger that this implies that while the teacher is being educated, the researcher is constructing new scientific knowledge. Perhaps it is because we believe that a necessary step in creating more inclusive mathematics education practices involves understanding how teaching and learning are mediated by our identities as physical, racial, ethnic, linguistic, gendered and social beings, that we believe it is critical to involve all those who participate in the educational process in fundamental research and to recognise and value their contributions in the construction of new knowledge.

Acknowledgments

We are grateful to the funding we have received from CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, Project No. 23038.019444/2009-33) in the course of this programme of research. Lulu Healy also received a grant from CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) during the writing of this paper. We would also like to thank the students, interpreters, teachers and researchers who collaborated with us during the design and execution of the research activities.

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